# Regular separability of WSTS

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**Regular separability of well-structured transition systems** (arXiv:1702.05334)
together with W. Czerwiński, S. Lasota, R. Meyer, K. N. Kumar, and P. Saivasan.
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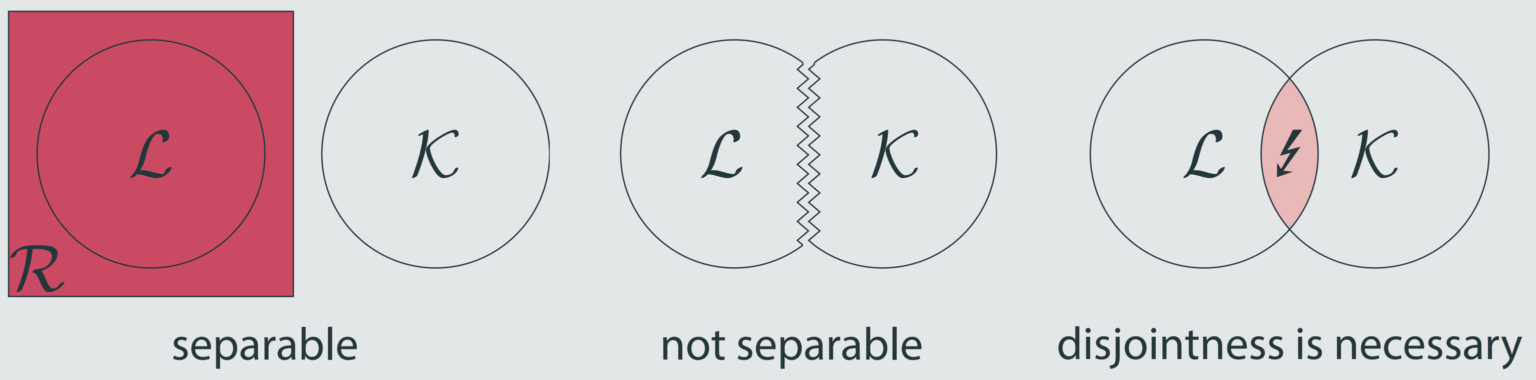
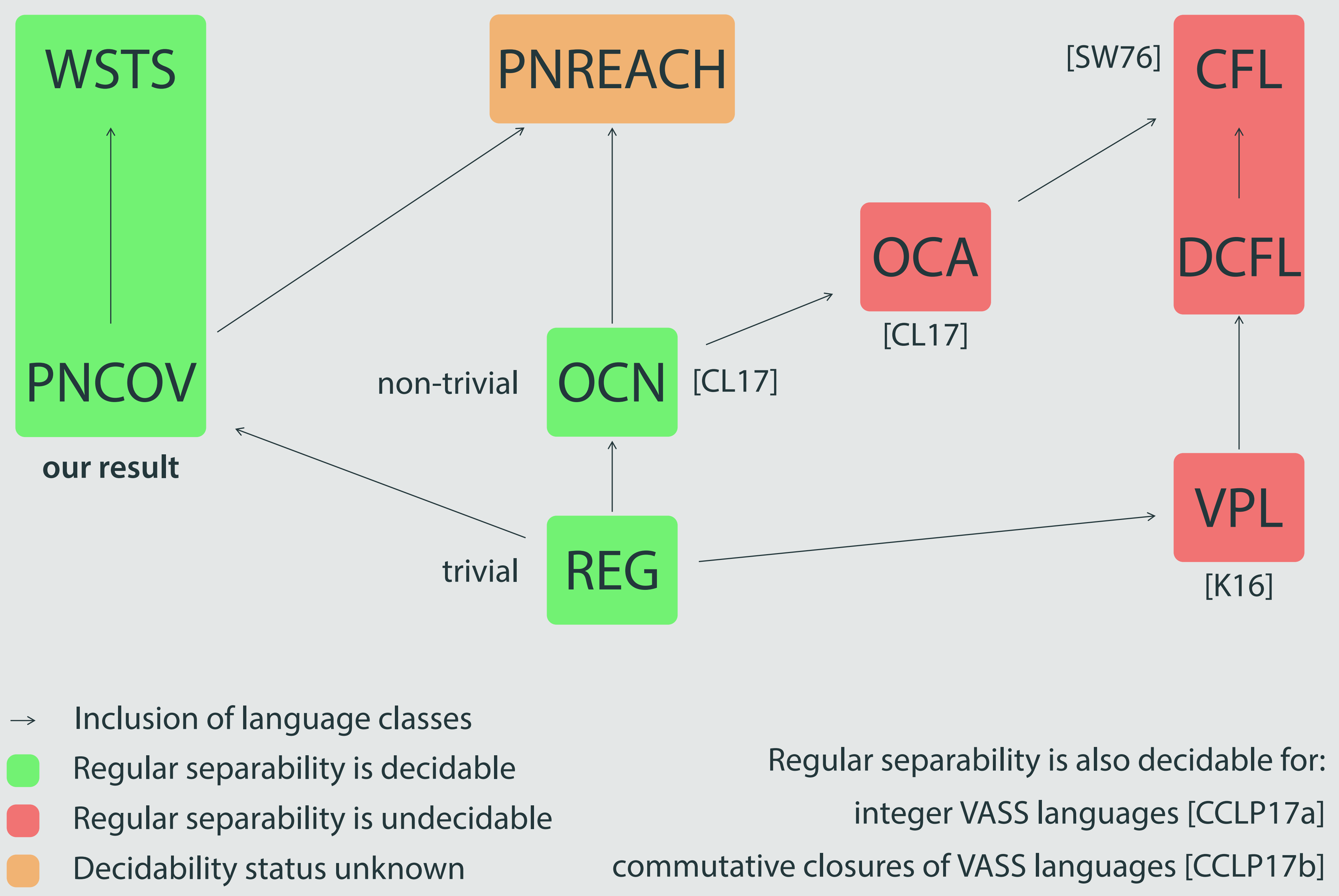
## Regular separability

**Regular separability of $\mathcal{F}$**
**Given:** Languages $\mathcal{L}, \mathcal{K} \subseteq \Sigma^*$ from class $\mathcal{F}$.
**Decide:** Is there $\mathcal{R} \subseteq \Sigma^*$ regular such that $\mathcal{L} \subseteq \mathcal{R}, \mathcal{K} \cap \mathcal{R} = \emptyset$?

**Intuition:**




**Related work:**




## Well-structured transition systems [FS01]

Labeled WSTS $\mathcal{W} = (S, \leqslant, T, I, F)$ over $\Sigma$ with
- $(S, \leqslant)$ well-quasi ordered states,
- $T \subseteq S \times \Sigma \times S$ transitions, (strongly) compatible with $\leqslant$,
- $I \subseteq S$ initial states,
- $F \subseteq S$ final states, upward closed.

**Coverability language:**

$$\mathcal{L}(\mathcal{W}) = \left\{ w \in \Sigma^* \;\middle|\; s_I \xrightarrow{w} s_F \text{ for some } s_I \in I, s_F \in F \right\}$$

**Examples:** Petri nets and extensions (transfer nets, reset nets, ...) with covering a marking as acceptance condition.

## Finite branching

$\mathcal{W}$ finitely branching if $I$ and $\text{Post}_\Sigma(s)$ finite for all $s \in S$.
$\mathcal{W}$ deterministic if $I$ and $\text{Post}_a(s)$ unique for all $s \in S, a \in \Sigma$.
$\mathcal{W}$ $\omega^2$-WSTS if $(S, \leqslant)$ does not embed the Rado order.

**Theorem:** *The following inclusions of language classes hold:*

lang. of $\omega^2$-WSTS $\subseteq$ lang. of deterministic WSTS,
lang. of fin.-branching WSTS $\subseteq$ lang. of deterministic WSTS.

## The result & its consequences

**Theorem:** *If two WSTS languages, one of them finitely branching, are disjoint, then they are regularly separable.*

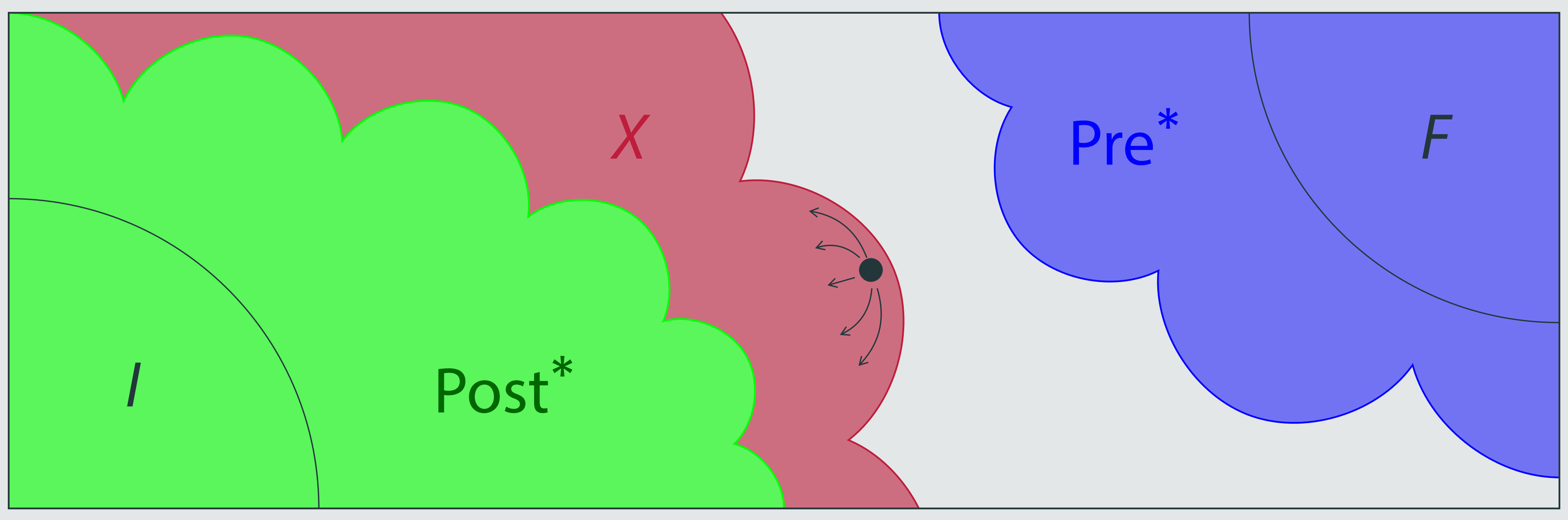
**Corollary:** *If a language and its complement are languages of finitely-branching WSTS, then they are necessarily regular.*

**Corollary:** *No subclass of the class of languages of finitely-branching WSTS beyond REG is closed under complement.*

**Proof approach:**
1. Show that finitely-represented inductive **invariants** can be turned into regular separators.
2. Show that such invariants always exist using **ideals**.

## 1. Invariants

Inductive invariant [MP95] $X$ for WSTS $\mathcal{W} = (S, \leqslant, T, I, F)$:
$X \subseteq S$ downward-closed, $I \subseteq X$, $F \cap X = \emptyset$, $\text{Post}_\Sigma(X) \subseteq X$.

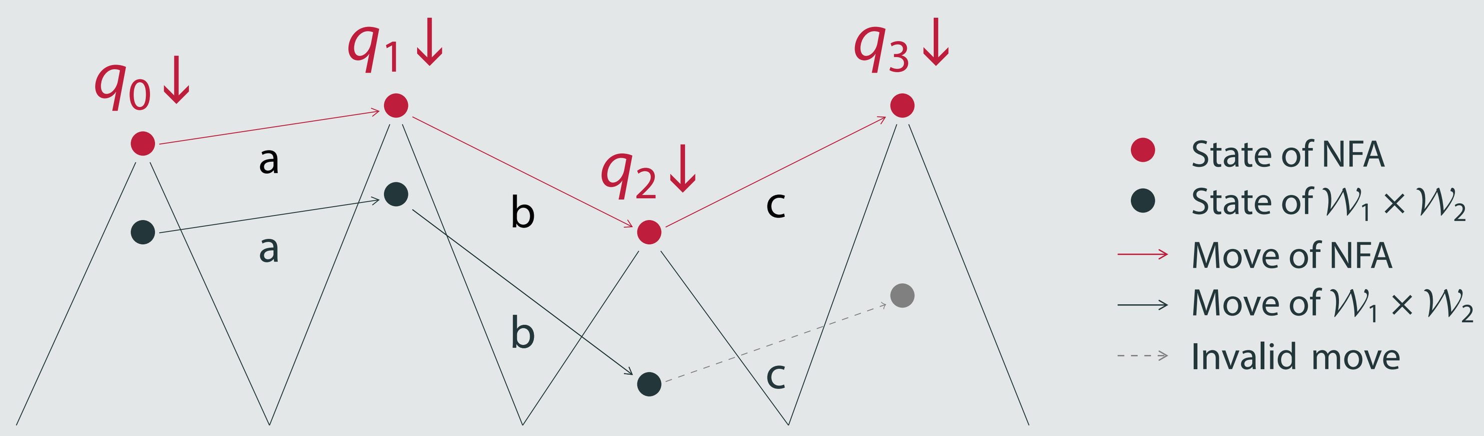



**Lemma:** $\mathcal{L}(\mathcal{W}) = \emptyset$ *iff inductive invariant for* $\mathcal{W}$ *exists.*

Call an invariant $X$ finitely represented if $X = Q{\downarrow}$ for $Q$ finite.

**Theorem:** *Let* $\mathcal{W}_1, \mathcal{W}_2$ *be WSTS,* $\mathcal{W}_2$ *deterministic. If* $\mathcal{W}_1 \times \mathcal{W}_2$ *has a finitely-represented inductive invariant, then* $\mathcal{L}(\mathcal{W}_1)$ *and* $\mathcal{L}(\mathcal{W}_2)$ *are regularly separable.*

**Proof:** Let $Q{\downarrow}$ be an invariant with $Q \subseteq S_1 \times S_2$ finite.
Construct NFA with states $Q$, accepting on $Q \cap F_1 \times S_2$.
NFA over-approximates the behavior of $\mathcal{W}_1 \times \mathcal{W}_2$:




Acceptance in NFA inherited from $\mathcal{W}_1 \implies \mathcal{L}(\text{NFA}) \subseteq \mathcal{L}(\mathcal{W}_1)$.
$Q \cap F_1 \times F_2 = \emptyset$, $\mathcal{W}_2$ deterministic $\implies \mathcal{L}(\text{NFA}) \cap \mathcal{L}(\mathcal{W}_2) = \emptyset$.

## 2. Ideals [KP92] [FG12, BFM14]

Let $\widehat{\mathcal{W}}$ be the ideal completion of $\mathcal{W}$. Note: $\mathcal{L}(\mathcal{W}) = \mathcal{L}(\widehat{\mathcal{W}})$.

**Proposition:** *If* $X$ *is an invariant for* $\mathcal{W}$*, then its ideal decomposition* $\text{Id-Dec}(X){\downarrow}$ *is a finitely-represented invariant for* $\widehat{\mathcal{W}}$.

Presented in Berlin in September 2018 at HIGHLIGHTS

# Regular separability of WSTS


### Abstract

We investigate the languages recognized by well-structured transition systems (WSTS). We show that, under mild assumptions, every two disjoint WSTS languages are regularly separable: There is a regular language containing one of them and being disjoint from the other. As a consequence, if a language as well as its complement are both recognized by WSTS, then they are necessarily regular. In particular, no subclass of WSTS languages beyond the regular languages is closed under complement.


### Publication


*Regular separability of well-structured transition systems.*
W. Czerwiński, S. Lasota, R. Meyer, S. Muskalla, K. N. Kumar, and P. Saivasan.
In CONCUR 2018, volume 118 of LIPIcs, pages 35:1–35:18, Schloss Dagstuhl, 2018.
The full version is available on arXiv, `arXiv.org/abs/1702.05334`.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