Regular separability of WSTS

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Publication

Regular separability of well-structured transition systems
with W. Czerwiński, S. Lasota, R. Meyer, K. N. Kumar, P. Saivasan.
CONCUR 2018. `arXiv:1702.05334`.

Regular separability

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Regular separability of $\mathcal{F}$

Given: Languages $\mathcal{L}, \mathcal{K} \subseteq \Sigma^*$ from class $\mathcal{F}$.

Decide: Is there $\mathcal{R} \subseteq \Sigma^*$ regular such that

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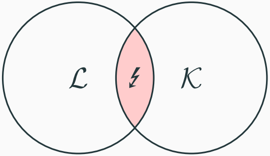

Disjointness is necessary!

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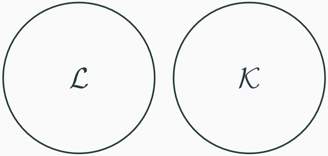
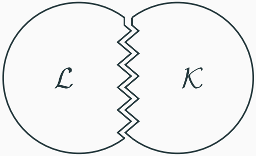


separable



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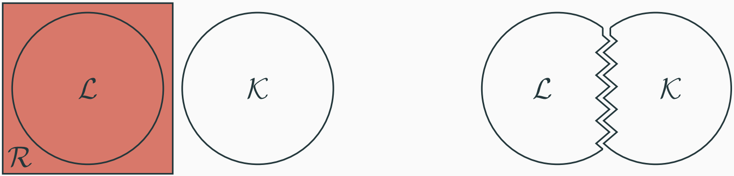
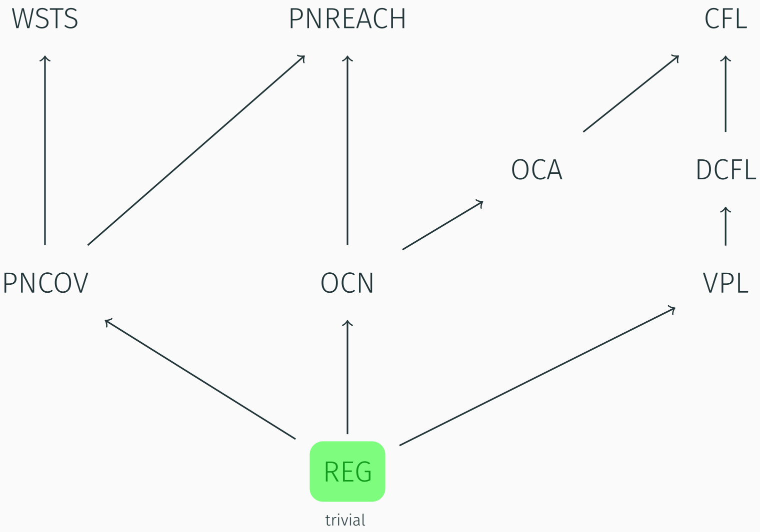


separable   not separable

Related work



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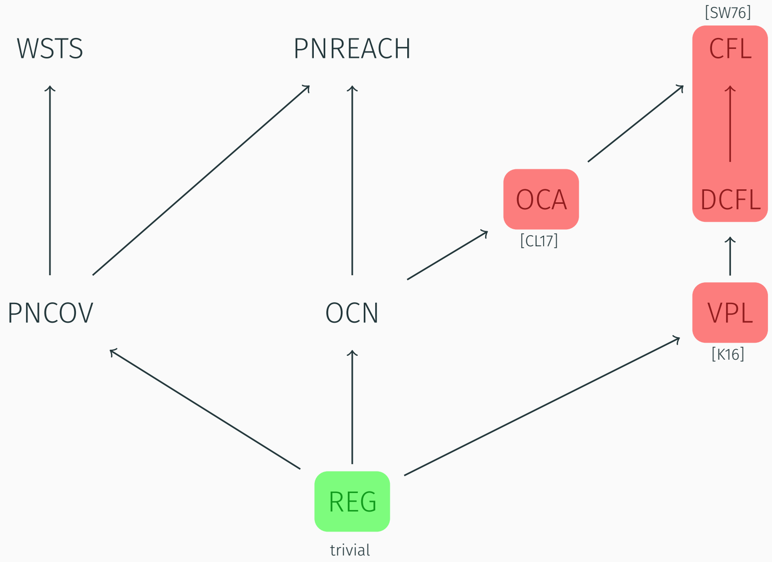

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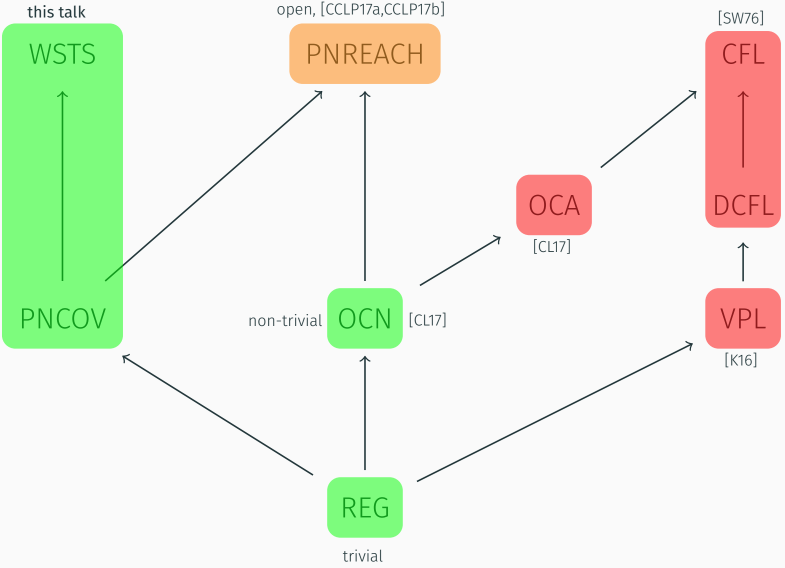

The result

Well structured transiton systems

Labeled well-structured transition systems (WSTS) [F87,ACJT96,FS01]

$$\mathcal{W} = (S, \leqslant, T, I, F)$$

- $(S, \leqslant)$ states, well-quasi ordered
- $T \subseteq S \times \Sigma \times S$ labeled transitions
- $I \subseteq S$ initial states
- $F \subseteq S$ final states, upward-closed
- (Strong) upward compatibility:

$$\begin{array}{ccc} s' & \overset{a}{\dashrightarrow} & r' \; (\exists) \\ \curlyvee\!| & & \curlyvee\!| \\ s & \overset{a}{\longrightarrow} & r \end{array}$$

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Coverability language

$$\mathcal{L}(\mathcal{W}) = \left\{ w \in \Sigma^* \;\middle|\; s_I \xrightarrow{w} s_F \text{ for some } s_I \in I, s_F \in F \right\}$$

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Examples:

- Petri nets with covering a marking as acceptance condition
- Transfer nets, reset nets, …
- Lossy channel systems

The result

Theorem

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Corollary

If a language and its complement are finitely-branching WSTS languages, then they are necessarily regular.

Generalizes earlier results for PNCOV [MKR98a,MKR98b]

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Corollary

No subclass of the class of languages of finitely-branching WSTS beyond REG is closed under complement.

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$\overset{(1)}{\subseteq}$ *Languages of finitely-branching WSTS*
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WSTS is ω^2 iff state space does not embed the Rado order.

(1) shows that result applies to all WSTS of practical interest.

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(2) proves that it is sufficient to show:

Theorem

If two WSTS languages, one of them deterministic, are disjoint, then they are regularly separable

Proof sketch

Proof approach

$$\mathcal{L}(\mathcal{W}_1), \mathcal{L}(\mathcal{W}_2) \text{ reg. sep} \overset{!}{\iff} \mathcal{L}(\mathcal{W}_1) \cap \mathcal{L}(\mathcal{W}_2) = \mathcal{L}(\mathcal{W}_1 \times \mathcal{W}_2) = \varnothing$$

$\mathcal{W}_1 \times \mathcal{W}_2$ has inductive invariant

Inductive invariant

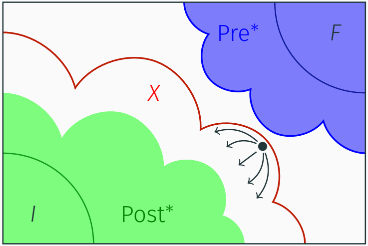
Inductive invariant [MP95] X for WSTS $\mathcal{W}$:

(1) $X \subseteq S$ downward-closed

(2) $I \subseteq X$

(3) $F \cap X = \varnothing$

(4) $\mathrm{Post}_{\Sigma}(X) \subseteq X$



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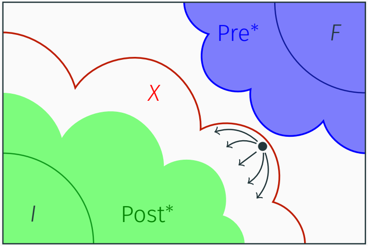

Lemma

$\mathcal{L}(\mathcal{W}) = \varnothing$ iff inductive invariant for $\mathcal{W}$ exists.

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$$\Updownarrow$$

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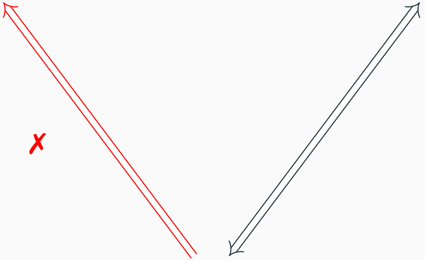

$\mathcal{W}_1 \times \mathcal{W}_2$ has inductive invariant

Finitely represented invariants

Call an invariant X finitely represented if $X = Q\downarrow$ for Q finite

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Theorem

Let $\mathcal{W}_1, \mathcal{W}_2$ WSTS, $\mathcal{W}_2$ deterministic.

If $\mathcal{W}_1 \times \mathcal{W}_2$ admits a finitely-represented inductive invariant, then $\mathcal{L}(\mathcal{W}_1)$ and $\mathcal{L}(\mathcal{W}_2)$ are regularly separable.

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Let $Q\downarrow$ be an invariant with Q finite.
Construct NFA with states Q.
NFA over-approximates $\mathcal{W}_1 \times \mathcal{W}_2$.

Proof approach

$$\mathcal{L}(\mathcal{W}_1), \mathcal{L}(\mathcal{W}_2) \text{ reg. sep} \overset{!}{\iff} \mathcal{L}(\mathcal{W}_1) \cap \mathcal{L}(\mathcal{W}_2) = \mathcal{L}(\mathcal{W}_1 \times \mathcal{W}_2) = \varnothing$$

✓

?

$\mathcal{W}_1 \times \mathcal{W}_2$ has fin.-rep. invariant

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✓ ✗

$\mathcal{W}_1 \times \mathcal{W}_2$ has fin.-rep. invariant

Ideals

Finitely represented invariants do not necessarily exist.

Solution: Ideals

Definition

For WSTS $\mathcal{W}$, let $\widehat{\mathcal{W}}$ be its ideal completion [KP92][BFM14,FG12]

Lemma

$\mathcal{L}(\mathcal{W}) = \mathcal{L}(\widehat{\mathcal{W}})$.

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Lemma

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Proposition

If X is an inductive invariant for $\mathcal{W}$,
then its ideal decomposition $\textsc{Idec}(X)\downarrow$
is a finitely-represented inductive invariant for $\widehat{\mathcal{W}}$.

Proof

Putting everything together:

Let $\mathcal{W}_1, \mathcal{W}_2$ be language-disjoint WSTS, $\mathcal{W}_2$ deterministic.

$\mathcal{W}_1 \times \mathcal{W}_2$ admits an invariant X.

Then $\textsc{Idec}(X){\downarrow}$ is a finitely-represented invariant for $\widehat{\mathcal{W}_1 \times \mathcal{W}_2} \cong \widehat{\mathcal{W}_1} \times \widehat{\mathcal{W}_2}$.

This gives rise to a regular separator.

Conclusion

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If two WSTS languages are disjoint,

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Can we drop the assumption of finite branching resp. ω^2 ?

Related problem:
Expressiveness of infinitely-branching non-ω^2 WSTS?

Thank you!

Questions?

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